

Assignment 5: Belief Functions

- 1 Consider an Expert System designed to detect and analyze airborne threats, where the frame of discernment $\Theta = \{f, e, c, n\}$. The intended interpretation is:

f	friendly aircraft
e	enemy aircraft
c	enemy missile - conventional warhead
n	enemy missile - nuclear warhead.

Suppose that belief functions are generated from a number of different sources:

m_1	from an advanced radar system
m_2	from a spy satellite system
m_3	from a computer-based intelligence system
m_4	from reports by the pilots of military aircraft.

Suppose also that at some point in time an incoming object is detected by advanced radar, traveling at high speed on a path potentially directed towards the White House. Interpretation of the radar signals might suggest a low belief that the object is an aircraft (friendly or enemy), a higher belief that the object is a missile (conventional or nuclear) together with some residual uncertainty (ignorance). Signals from an overhead spy satellite might suggest, based on trajectory and velocity, that the object is definitely a missile, with a low belief that it is conventional, a higher belief that it is nuclear, and some belief distributed between the two options (with some residual ignorance based on potential unreliability of the equipment). Military intelligence, however, might suggest that the country of origin is without nuclear capability; in conjunction with a certain modest recognition of past failings, this might be represented by a small belief that the object is, in fact, a friendly aircraft, a larger belief that it is an unfriendly projectile (whether aircraft or missile), and some belief that it must at least be one or other of these as it cannot possibly be nuclear (along with residual uncertainty naturally accruing to military intelligence processes!). Finally, however, suppose the pilot of a high speed interceptor plane reports that the object looks like a friendly aircraft (but might not be, as it is a bright sunny day and difficult to see the projectile clearly).

A set of belief functions corresponding to this scenario might be given by the following basic probability assignments:

$$m_1(\{f, e\}) = 0.2, m_1(\{c, n\}) = 0.75, m_1(\{f, e, c, n\}) = 0.05 \quad (1)$$

$$m_2(\{c\}) = 0.3, m_2(\{n\}) = 0.6, m_2(\{c, n\}) = 0.05, m_2(\{f, e, c, n\}) = 0.05 \quad (2)$$

$$m_3(\{f\}) = 0.1, m_3(\{e, c\}) = 0.6, m_3(\{f, e, c\}) = 0.25, m_3(\{f, e, c, n\}) = 0.05 \quad (3)$$

$$m_4(\{f\}) = 0.6, m_4(\{f, e, c, n\}) = 0.4. \quad (4)$$

This evidence is less than ideal. Quite apart from the fact that the different sources of evidence do not appear to be sending consistent signals, none of the individual belief functions except the last is consonant. m_1 , m_2 and m_3 are separable support functions, and m_4 is simple.

- (a) Combine these four belief functions using Detspiter's Rule. Of course, since this rule is associative and commutative, it does not matter in what order you do this. However, to simplify checking of intermediate results during grading, please combine these pairwise in the order given; i.e., combine m_1 and m_2 , then combine the results with m_3 , and finally combine these results with m_4 .
- (b) Convert these four basic probability assignments to commonalities, combine the commonalities by pointwise multiplication (of course, you can now combine all four at once and do not need to do this pairwise), and convert the results back to a basic probability assignment. Be sure that you can complete this final step correctly without relying on the fact that the answer should be the same as for part (a)!
- (c) In the light of this combined evidence, what is the final Belief (expressed as a Bel function) that the White House is under attack by a missile?
- (d) What is the Plausibility of a missile attack?
- (e) What is the final Belief in a nuclear missile attack?
- (f) What is the Plausibility of a nuclear missile attack?
- (g) Which of the four individual possible outcomes is the most plausible explanation?
- (h) Determine the canonical decomposition of each of the original belief functions into simple support functions.
- (i) Two of the resulting decompositions are double combinations, and the third involves a single combination. For each of these five combinations, compute just the normalizing constants. Then compute the product of these five constants together with the three normalizing constants used in part (a). Finally, compute $(1 - \text{the result})$. The quantity you have now calculated is Shafer's measure of the conflict in the final belief function. It represents the proportion of the total belief treated as "noise" during combination because it was assigned to the empty intersection.
- (j) Finally, for the combined belief function, compute the quantities:

$$N(m) = \sum_{A \subseteq \Theta} m(A) \cdot \log_2(|A|) \quad \text{and} \quad E(m) = - \sum_{A \subseteq \Theta} m(A) \cdot \log_2(Pl(A))$$

representing non-specificity and dissonance respectively. Since each of these has range $[0, \log_2 |\Theta|]$, normalize them by dividing by 2. We will discuss interpretations of these measures in class next time.