26:198:722 Expert Systems

- Mid-Term Examination
- Possibility Theory
- Spohn's Epistemic Calculus
- Belief Functions and Assignment 5

Mid-Term Examination

?

Possibility Theory

- Possibility Theory is a computational implementation (largely inspired by Dubois and Prade) of Zadeh's Fuzzy Logic
- Shenoy showed that it may be re-formulated as a variant of Valuation Networks, so that possibilities may be propagated in Join Trees using the Shenoy-Shafer algorithm

Possibility Theory

- A possibility function is a function

  $\pi : 2^{\Omega_p} \rightarrow [0,1]$

  such that:

  $\exists x \in \Omega_p : \pi(x) = 1$

  and

  $\forall a \in 2^{\Omega_p} : \pi(a) = \max\{\pi(x) | x \in a\}$

Possibility Theory

- By virtue of this second condition, possibility functions are completely determined by their values for singleton elements
- Intuitively, the degree of possibility for a subset a is $1-\pi(\sim a)$
  and the degree of impossibility is $1-\pi(a)$

Possibility Theory

- Marginalization

$$\forall y \in \Omega_{a-\{X\}} : \pi^{\downarrow a-\{X\}}(y) = \max\left\{\pi(y,x) \middle| x \in \Omega_X\right\}$$

- Combination

$$\pi_1 \otimes \pi_2(x) = \begin{cases} K^{-1}\pi_1\left(x^{\downarrow a}\right)\pi_2\left(x^{\downarrow b}\right) & K \neq 0 \\ 0 & K = 0 \end{cases}$$

where $K = \max\left\{\pi_1\left(x^{\downarrow a}\right) \cdot \pi_2\left(x^{\downarrow b}\right) \middle| x \in \Omega_{a\cup b}\right\}$

Possibility Theory

Example: Marginalization

Objective 1	**Objective 2**	**Possibility**
true	true	1
true	false	.5
false	true	.3
false	false	.1

Possibility Theory

Example: Marginalization

Objective 1 Possibility

true	1
false	.3

Objective 2 Possibility

true	1
false	.5

Possibility Theory

- So for Objective 1, the degree of possibility for “true” is 0.7, and the degree of impossibility is 0
- Similarly, for Objective 2, the degree of possibility for “true” is 0.5, and the degree of impossibility is 0

Possibility Theory

Example: Combination

Objective 1	**Possibility**
true	1
false	.4

Objective 1	**Possibility**
true	1
false	.6

Possibility Theory

Example: Combination

Objective 1	**Possibility**
true	1
false	.24

Possibility Theory

Example: Combination

Objective 1	**Possibility**
true	1
false	.4

Objective 1	**Possibility**
true	.6
false	1

Possibility Theory

Example: Combination

Objective 1 Possibility

true	.6
false	.4

Objective 1 Possibility

true	1
false	.67

Possibility Theory

- We can represent complete ignorance using possibilities as follows:

Objective 1	**Objective 2**	**Possibility**
true	true	1
true	false	1
false	true	1
false	false	1

Possibility Theory

- We can define logical relationships such as “AND” nodes as follows: A= B&C:

A	**B**	**C**	**Possibility**
a	b	c	1
a	b	~c	0
a	~b	c	0
a	~b	~c	0
~a	b	c	0
~a	b	~c	1
~a	~b	c	1
~a	~b	~c	1

Possibility Theory

- Discounted “AND” nodes as discussed for probabilities and belief functions CANNOT be defined
- Dubois and Prade argue that statistical sampling is contrary to the non-probabilistic nature of possibility theory: however, it seems that it could be incorporated using normalized maximum likelihood functions

Possibility Theory

- Joint possibilities generally cannot be uniquely determined from marginals, as for probabilities and belief functions:

X	**Y**	**P1**	**P2**	**P3**
x	y	1	1	1
x	~y	.4	.4	.4
~x	y	.2	.2	0
~x	~y	.2	.1	.2

  have the same marginals for X and Y.
- However, when combined with an "AND" node, all three produce the same results!!!

Possibility Theory

- When multiple nodes are combined in an "AND" node, the effect is that the degree of possibility for the conjunction is equal to the degree of possibility for the least possible conjunct
- This is significantly different from probabilities and belief functions, and is very relevant, for example, to auditing

Spohn's Epistemic Calculus

- Spohn introduced his theory of epistemic states in order to represent plain human beliefs in a non-probabilistic way easily amenable to revision
- Initially they were based on functions mapping into the ordinals; later he changed this to the natural numbers

Spohn's Epistemic Calculus

- For technical reasons, we will define disbeliefs as functions mapping to the natural numbers extended by adding a representation of infinity, ∞

Spohn's Epistemic Calculus

- A disbelief function is a function

$$\delta : 2^{\Omega_d} \rightarrow N^{+}$$

such that:

$$\exists x \in \Omega_p : \pi(x) = 0$$

and

$$\forall a \in 2^{\Omega_d} : \delta(a) = \min\{\pi(x) | x \in a\}$$

Spohn's Epistemic Calculus

- By virtue of this second condition, disbelief functions are completely determined by their values for singleton elements
- Intuitively, the degree of disbelief for a subset a is $\delta(a)$
  and the degree of belief is $\delta(\sim a)$

Spohn's Epistemic Calculus

- Marginalization

$$\forall y \in \Omega_{a-\{X\}} : \delta^{\downarrow a-\{X\}}(y) = \min\left\{\delta(y,x) \middle| x \in \Omega_X\right\}$$

- Combination

$$\delta_1 \otimes \delta_2(x) = \delta_1\left(x^{\downarrow a}\right) + \delta_2\left(x^{\downarrow b}\right) - K$$

where $K = \min\left\{\delta_1\left(x^{\downarrow a}\right) + \delta_2\left(x^{\downarrow b}\right) \middle| x \in \Omega_{a\cup b}\right\}$

Spohn's Epistemic Calculus

Example: Marginalization

Objective 1	**Objective 2**	**Disbelief**
true	true	0
true	false	5
false	true	7
false	false	9

Spohn's Epistemic Calculus

Example: Marginalization

Objective 1	**Disbelief**
true	0
false	7
Objective 2	**Disbelief**
true	0
false	5

Spohn's Epistemic Calculus

Example: Combination

Objective 1	**Disbelief**
true	0
false	6

Objective 1	**Disbelief**
true	0
false	4

Spohn's Epistemic Calculus

Example: Combination

Objective 1	**Disbelief**
true	0
false	10

Spohn's Epistemic Calculus

Example: Combination

Objective 1	**Disbelief**
true	0
false	6
Objective 1	**Disbelief**
true	4
false	0

Spohn's Epistemic Calculus

Example: Combination

Objective 1	**Disbelief**
true	4
false	6
Objective 1	**Disbelief**
true	0
false	2

Spohn's Epistemic Calculus

- We can represent complete ignorance using disbeliefs as follows:

Objective 1	**Objective 2**	**Disbelief**
true	true	0
true	false	0
false	true	0
false	false	0

Spohn's Epistemic Calculus

- We can define logical relationships such as "AND" nodes as follows: A= B&C:

A	**B**	**C**	**Disbelief**
a	b	c	0
a	b	~c	∞
a	~b	c	∞
a	~b	~c	∞
~a	b	c	∞
~a	b	~c	0
~a	~b	c	0
~a	~b	~c	0

Spohn's Epistemic Calculus

- Discounted "AND" nodes as discussed for probabilities and belief functions CANNOT be defined
- Statistical sampling is contrary to the intended ordinal nature of epistemic (dis)beliefs

Spohn's Epistemic Calculus

- Joint disbeliefs generally cannot be uniquely determined from marginals, as for probabilities and belief functions:

X	**Y**	**P1**	**P2**	**P3**
x	y	0	0	0
x	~y	4	4	4
~x	y	7	7	11
~x	~y	7	74	7

  have the same marginals for X and Y.
- However, when combined with an "AND" node, all three produce the same results!!!

Spohn's Epistemic Calculus

- When multiple nodes are combined in an "AND" node, the effect is that the degree of belief for the conjunction is equal to the degree of belief for the least believed conjunct
- This is significantly different from probabilities and belief functions, and is very relevant, for example, to auditing

Spohn's Epistemic Calculus

- In addition, Spohn's system offers the prospect of elicitation of ordinal rankings rather than highly sensitive real values

Belief Functions

?